Lande's g factor in case of jj- coupling

In jj- coupling the s^* and l^* of each electrons are coupled together and quantized to form their own resultant j^*. With two electrons we may write $l_1^* + s_1^* = j_1^*$ and $l_2^* + s_2^* = j_2^*$

The resultant vectors j_1^* and j_2^* are quantized separately with the magnetic moments

$$\mu_1 = g_1 j_1^* \frac{eh}{4\pi m_0} \text{ and } \mu_2 = g_2 j_2^* \frac{eh}{4\pi m_0} \quad (1)$$

The j_1^* and j_2^* is then coupled together and quantized to form a resultant J^*, these moments are projected on J^* and added to give the total magnetic moment of the atom,

$$\mu_J = g_1 j_1^* \frac{he}{4\pi m_0} \cos(j_1^* J^*) + g_2 j_2^* \frac{he}{4\pi m_0} \cos(j_2^* J^*) \quad (2)$$

$$= \left[g_1 j_1^* \cos(j_1^* J^*) + g_2^* j_2^* \cos(j_2^* J^*) \right] \frac{eh}{4\pi m_0} \quad (3)$$

Replacing the terms in the bracket by g times J^*, we get

$$g \cdot J^* = g_1 j_1^* \cos(j_1^* J^*) + g_2 j_2^* \cos(J_2^* J^*) \quad (4)$$

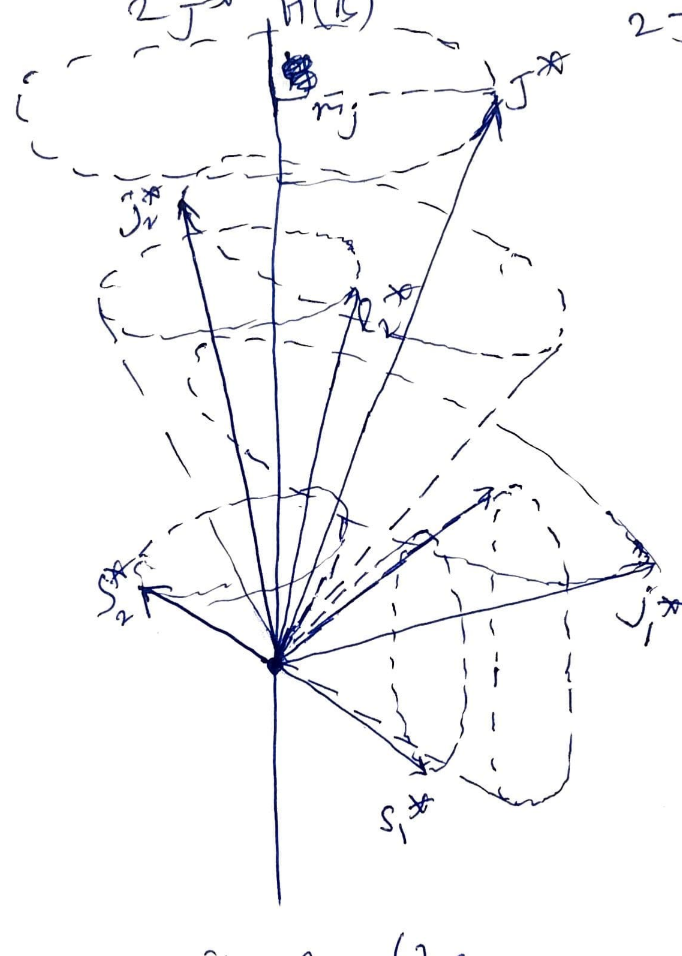
Since the angles between j_1^*, j_2^* and J^* are constant, the cosine formula gives,

$$j_1^* \cos(j_1^* J^*) = \frac{J^{*2} + j_1^{*2} - j_2^{*2}}{2J^*}$$

$$j_2^* \cos(j_2^* J^*) = \frac{J^{*2} + j_2^{*2} - j_1^{*2}}{2J^*} \quad -(5)$$

$\Rightarrow$ the g-factor for jj-coupling now becomes

$$g = g_1 \frac{J^{*2} + j_1^{*2} - j_2^{*2}}{2J^{*2}} + g_2 \frac{J^{*2} + j_2^{*2} - j_1^{*2}}{2J^{*2}} \quad -(6)$$



jj-coupling

The Zeeman Effect - Two electron system

In a weak magnetic field the atom as a whole is quantized with B in such a fashion that the projection of the angular momentum $J^* h/2\pi$ on B is equal to $M_J h/2\pi$.

Where M_J takes values differing from each other by unity, from $M_J = +J$ to $M_J = -J$.

In terms of the vector model these quantum conditions are expressed as

$$J^{*} \cos(J^{*}B) = M_J \quad \text{where } M_J = 0, \pm 1, \pm 2 \ldots \pm J \qquad (1)$$

On the classical model of a precessing atom, J^{*} carries out a Larmor precession around B with an angular velocity given by

$$\omega_L = B \cdot g \cdot \frac{e}{2m_0} \qquad (2)$$

Multiplying by the projection of the mechanical moment on B, the energy of this precession

$$\Delta E = B \cdot g \cdot \frac{e}{2m_0} J^{*} \cdot \frac{h}{2\pi} \cos(J^{*}B) \qquad (3)$$

In terms of the magnetic quantum number

$$\Delta E_{M_J} = M_J \cdot g \cdot B \frac{eh}{4\pi m_0} \qquad (4)$$

The energy in wave number is given by

$$\Delta \nu = g \frac{eB}{4\pi m_0} \cdot \Delta M_J \qquad (5)$$

$$\Delta \bar{\nu} = g \, \Delta M_J \cdot L \quad \text{cm}^{-1} \qquad (6)$$

$$L = \text{Lorentz unit} = \frac{eB}{4\pi m_0 c}$$

If $T \to$ the term value of a field-free energy level, then in a weak magnetic field the term value of each magnetic level will be

$$T_M = T - M_J \cdot g \frac{Be}{4\pi m_0 c} \quad \text{cm}^{-1} \qquad (7)$$

On the classical model for L.S-coupling it was assumed (1) that s_1^* and s_2^* precess rapidly around S^*, (2) that l_1^* and l_2^* precess rapidly around L^*, (3) that L^* and S^* precess more slowly around J^* and (4) that J^* in turn precesses still more slowly around B.

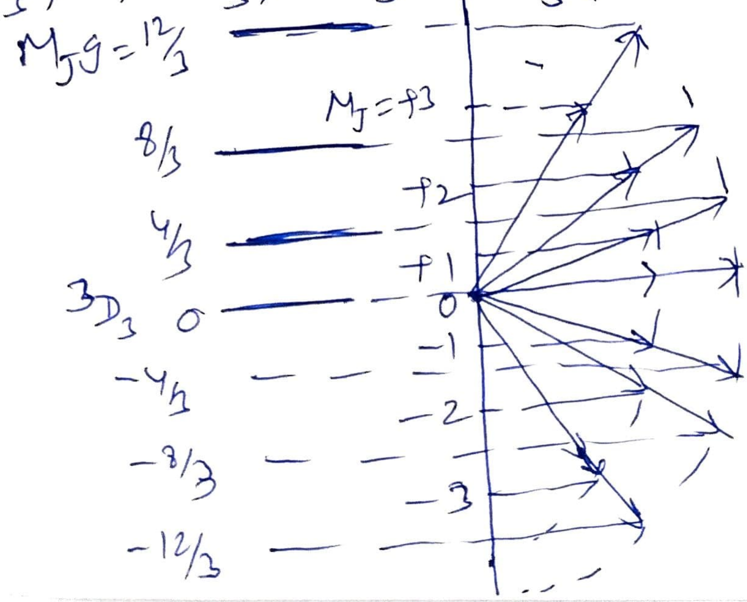
As an example of the splitting up of an energy level in a magnetic field, consider the case of $^3D_3$ level. The value of Lande's g factor is

$$g = 1 + \frac{J^{*2} + S^{*2} - L^{*2}}{2J^{*2}}$$

$$= 1 + \frac{3\cdot 4 + 1\cdot 2 - 2\cdot 3}{2\cdot 3\cdot 4}$$

$$g = 1 + \frac{1}{3} = \frac{4}{3}$$

With $g = \frac{4}{3}$ and $J = 3$, there are seven equally spaced magnetic levels $M_J = 3, 2, 1, 0, -1, -2, -3$, shifted from the field-free level by $M_J \cdot g = \frac{12}{3}, \frac{8}{3}, \frac{4}{3}, 0, -\frac{4}{3}, -\frac{8}{3}, -\frac{12}{3}$.

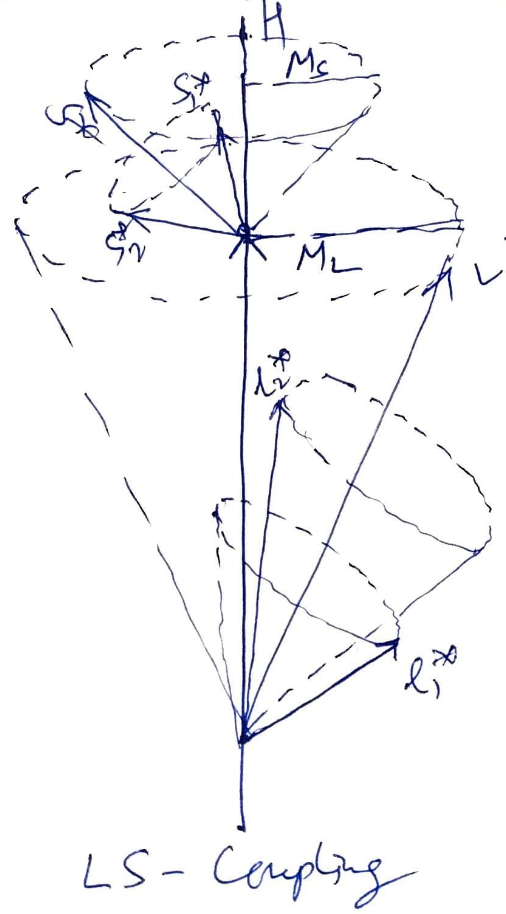


Splitting of $^3D_3$ level in a weak magnetic field

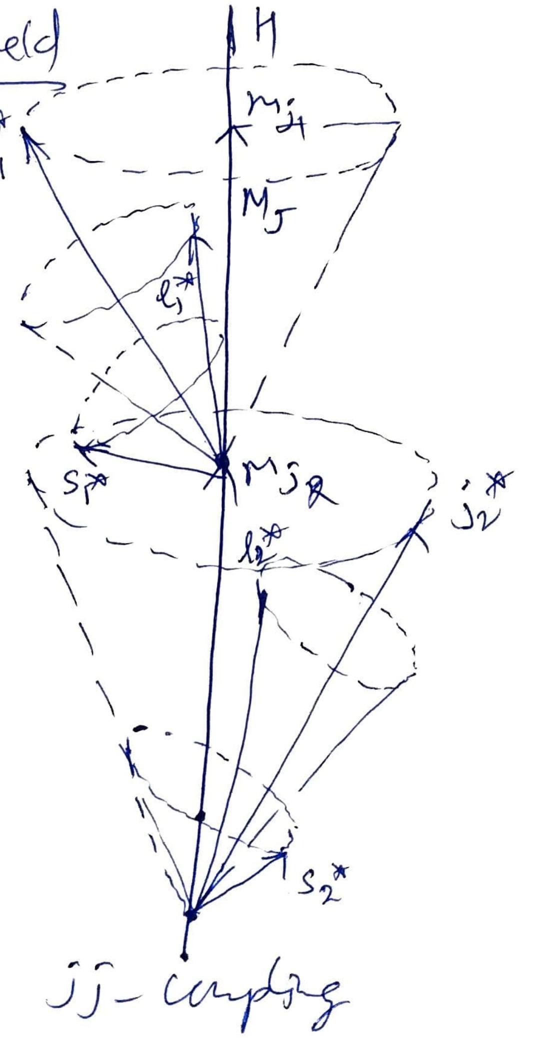
Paschen-Back Effect :- If the magnetic field is continually increased → the interaction energy between J^* and B becomes so great that the coupling between S^* and L^* in the case of LS-coupling, or between j_1^* and j_2^* in the case of jj-coupling is broken. As the field is still further increased, S^* and L^*, or j_1^* and j_2^* will independently be quantized with the field B. ⟶ Paschen-Back effect.

L^* and S^* precessing independently around the field direction, J^* is no longer constant in magnitude and ceases to be a quantum number. Similarly with j_1^* and j_2^* precessing independently around B, their resultant J^* ceases to have any meaning.

Strong field



LS - Coupling



jj - coupling